The GZK Bound in Discrete Space

N. Kersting
Department of Physics
Sichuan University
Chengdu, 610065 P.R. China
kest@mail.tsinghua.edu.cn

SCUPHY-TH-06001

Abstract

The maximum distance bound for ultrahigh energy cosmic rays (UHECR) with energies above the Greisen-Zatsepin-Kuzmich cutoff $\sim 10^{19}$ eV *relaxes significantly if there is some mechanism that forces UHECR to propagate in discrete intervals, rather than continuously, through intergalactic space. In particular, intervals as small as a femtometer relax the bound for protons by an order of magnitude and potentially account for the observed excess of UHECR flux.*

Introduction

Recent experiments [1, 2, 3] have indicated that the flux of cosmic rays entering the Earth's atmosphere with energy beyond the Greisen-Zatsepin-Kuzmich (GZK) limit [4] of roughly $5 \cdot 10^{19}$ eV is much higher than expected. According to the original papers on the subject, such ultra-high-energy cosmic rays (UHECR), presumably consisting of photons, nucleons, or heavier nuclei, must be rapidly attenuated after travelling a distance of $O(10)$ Mpc through intergalactic space due to interactions such as $N + \gamma \rightarrow \Delta \rightarrow N + \pi$ and $\gamma + \gamma \rightarrow e^+e^-$ with the cosmic microwave background radiation (CMBR), and since there are no known sources of UHECR within this radius of the Earth the observed flux in our atmosphere should be negligable. If further experimentation verifies excess in UHECR flux then the simple GZK model may not be the correct theory of UHECR propagation in space.

The excess of UHECR has been known for at least a decade [5], but of interest lately is analysis showing significant clustering of UHECR events in the sky in directions that point[1] to candidate sources 140 MpC away [7]. The possibility that UHECR are able to travel such long distances through the CMBR lends strong motivation to revise the GZK bound.

The original derivation of the GZK bound for protons depends on the energy of the protons (E), the energy loss per interaction (ΔE) with a CMBR photon, the number density of CMBR photons (n), and the interaction cross-section (σ) as follows:

$$\lambda_{GZK} = (\frac{E}{\Delta E})(\frac{1}{n\sigma}) \tag{1}$$

For example, for $E = 10^{20}$ eV protons incident on CMBR photons giving resonant pion-production we have $\sigma \approx 200$ μb, $n \approx 400$ cm^{-3}, and therefore $\lambda_{GZK} \approx 10^{24}$ $m \approx 30$ Mpc (one can derive a similar limit for photons). The dilemma is that no potential UHECR sources, such as Active Galactic Nuclei, are observed to be within that radius of the Earth [8].

Models to resolve the dilemma usually introduce additional assumptions to modify ΔE or σ in (1), resulting in a larger λ_{GZK}. For example, ΔE can decrease in theories in which energy conservation fails at high energies [9]; σ decreases in models where the UHECR is initiated by an exotic particle or neutrino which interacts very weakly with the CMBR [10, 11].

The current Letter proposes that one can extend the GZK bound if UHECR propagate in discrete jumps through space-time, rather than in a continuous path. It turns out that the form of the second factor in (1) changes, and a jump size as small as a few femtometers is sufficient to resolve the dilemma. In contrast to the abovementioned theories, this model introduces no new

[1] These directions are meaningful because intergalactic magnetic fields typically bend the paths of UHECR by less than a degree [6]

particles or violation of conservation principles, but rests merely on the hypothesis that spacetime is discrete.

Calculation

First let us recall the derivation of the second factor in parenthesis in (1), the mean free path length for, say, a proton. This comes about from a pseudo-classical approximation of the proton-CMBR interaction, justified by the relative diffuseness of the CMBR. As shown in Figure 1, from the perspective of the CMBR the proton sweeps out an effective tube in 3-space of cross-sectional area σ and length R; the probability of interaction with a CMBR photon along this path is just the volume swept out, σR, divided by the photon number density n. Therefore the probability of interaction per unit path length is constant, and its inverse, the mean free path of the proton, is $(n\sigma)^{-1}$.

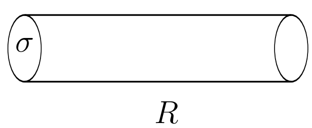

Figure 1. *Segment of UHECR path*

Now suppose the proton doesn't move in a continuous line, but propagates in jumps[2] of size R. Working in the same pseudo-classical approximation above, in the course of one jump the proton occupies an approximate volume of $\sigma^{3/2}$ in the CMBR, before jumping again. The mean path length (though this 'path' is traced out over many jumps) is then $\frac{R}{n\sigma^{3/2}}$. Since the fractional energy lost per interaction remains the same, the GZK bound should be replaced by

$$\lambda_R = (\frac{E}{\Delta E})(\frac{R}{n\sigma^{3/2}}) \tag{2}$$

and depends on the size R of the proton jumps. Note that as $R \to \sqrt{\sigma}$, λ_R becomes λ_{GZK}, and for $R < \sqrt{\sigma}$ λ_{GZK} replaces λ_R.

Using again the example of photopion production, with $R \approx 1\ fm$, gives $\lambda_R = 300\ MpC$. Remarkably, if UHECR protons jump in intervals as small as a femtometer they can easily traverse the intergalactic space to the nearest UHECR sources 140 MpC away.

[2]Observable properties of the proton such as velocity, energy, etc. are then defined as suitable averages over many jumps.

Comments

Supposing UHECR protons do jump in fermi-size steps through intergalactic space, an immediate concern is whether this behaviour applies to other energies and other particles: i.e. few researchers would object to discreteness at the Planck-scale, but do existing observations rule out spacetime discreteness at the fermi-scale? One expects physics of very low energies (eV- and below) to be insensitive to motions on the order of a fermi. However, already in the realm of medium- to high-energies($E > MeV$) severe constraints may arise in more sensitive environments, e.g. stellar interiors or particle detectors in accelerator-based experiments, though these are not nearly as uniform and diffuse as the CMBR and hence require a more sophistcated treatment than that appearing in this Letter.

In the event, however, in which the discretization of space is a dynamic phenomenon depending on energy (and possibly other quantum numbers) these phenomenological constraints may be weakened or removed, and investigations in this direction are underway [12].

References

[1] Hayashida, N., et al. AGASA collaboration., Astropart. Phys. **10**:303 (1999).

[2] Bird, D. J., et al. HIRES collaboration., Astrophys. J. **511**:739 (1999).

[3] Bellido, J. A. et al. , Astropart. Phys. **15**:167 (2001).

[4] K. Greisen, Phys. Rev. Lett. **16**:748 (1966); G. T. Zatsepin and V. A. Kuzmin, Sov. Phys. JETP Lett. **4**:78 (1966).

[5] D. Bird et al., Phys. Rev. Lett. **71**:3401 (1993); Astrophys. J. **424**:491 (1994)

[6] Dolag, K., Grasso, D., Springel, V., and Tkachev, I., JCAP **0501**:009 (2005)

[7] Farrar, G.R., arXiv:astro-ph/0501388

[8] Olinto, A.G. AIP Conf.Proc.**745**:48-59 (2005); Tinyakov, P. and Tkachev, I., arXiv:hep-ph/0212223.

[9] Ellis, J. et al. Phys.Rev.**D63**:124025 (2001); Coleman, S. and Glashow, S. L., arXiv:hep-ph/9808446

[10] Cskai, C. et al. JCAP **0305**:005 (2003); Berezinsky V. et al. Phys. Rev. Lett. **79**:4302 (1997);

[11] Jain, P. et al. Phys.Lett.**B484**:267-274 (2000); Kachelriess, M. and Plumacher, M. Phys.Rev.**D62**:103006 (2000)

[12] Kersting, N. work in progress